What is a Qubit?

Qu**antum **Bi**nary Dig**it

Caroline S. Gorham, March 19 2019

Past: Classical Computation

- Classical computer *central processing units* (CPU) represent data using a binary (i.e., two-symbol) system.

- *A bit is two possible states of transistor on/off.*

strings of bits (either 0 or 1) represent data

…	1	0	0	1	1	0	1	1	0	1	0	1	0	…

a byte is a string of 8 bits.

This can represent $2^8 = 256$ combinations, a wide variety of data.

In ASCII binary code, "wikipedia" (9-letters) is written in 9 bytes.

W	i	k	01010111	01101001	01101011
i	p	e	01101001	01110000	01100101
d	i	a	01100100	01101001	01100001

Storing of information does not scale with the number of bits, and calculations are done in the same way as by hand.

Future: What is Quantum Computation ?

A quantum computer stores data using *qu*bits where the 'binary' system is now quantum-mechanical ($|0>$ or $|1>$).

That is, in a *qubit*, there are also two states – a *two-level system (TLS).*

Being a quantum mechanical system, qubits can exhibit many interesting effects that are not possible in CPUs (e.g., superposition, entanglement of multiple qubits). This generates an exponentially larger computational space than CPUs!!

An example of the implementation of a qubit TLS could be particles with two spin states: "down" ($|\downarrow\rangle$). and "up"($|\uparrow\rangle$).

$$|\psi\rangle = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = c_1|\uparrow\rangle + c_2|\downarrow\rangle \qquad \text{where, } |\uparrow\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ and } |\downarrow\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

individual ions trapped in a vacuum using electric and magnetic fields are also TLS

Superconducting qubit: *Josephson junctions (JJs)*

The most popular implementation of qubits are *superconducting*.

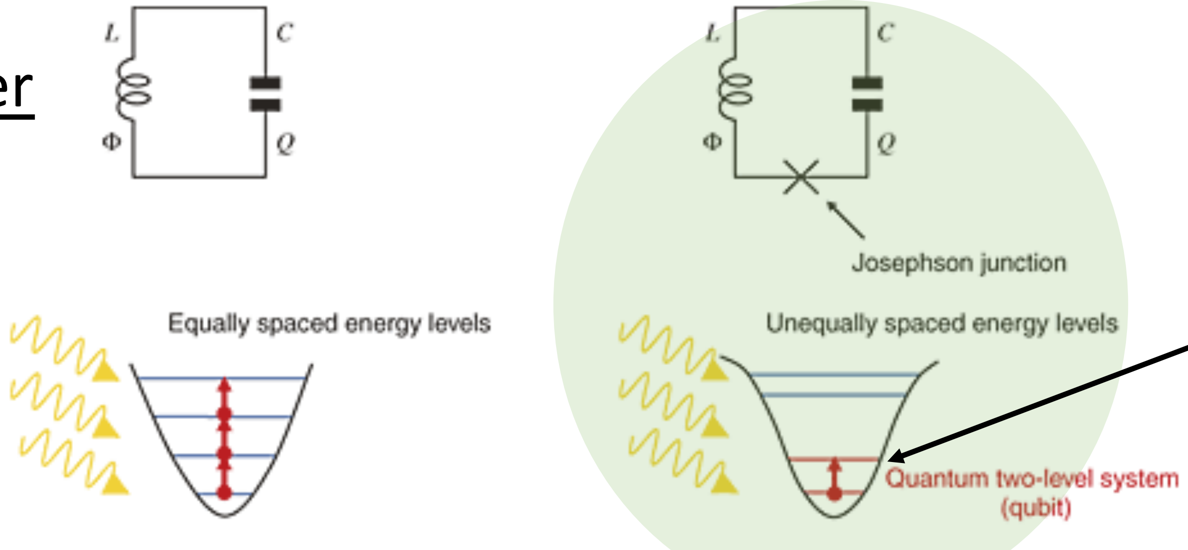
This is because superconductivity is a *macroscopic quantum phenomenon*.

Complex order parameter

$$\psi = |\psi| e^{i\theta}$$

Heisenberg Uncertainty Relation

$$\Delta|\psi|\Delta\theta \geq 1$$



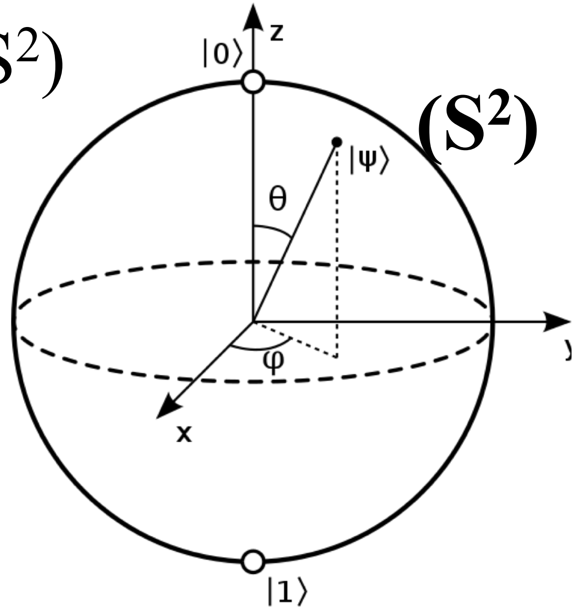
(a) LC-circuit without Josephson junction (b) LC-circuit with Josephson junction

First two energy levels are *isolated*, forming **TLS**!

TLS can be isolated by implementing *Josephson junctions (JJs)*, which are non-linear inductors that accumulate (magnetic field) energy when current passes through it.

Representing *qu*bit state on a Bloch sphere (S^2)

TLS qubits have a simple geometric picture, because pure states can be identified with points on the surface of a Bloch sphere (S^2)



__The total probability of a QM wavefunction has to be 1.__ $\langle\psi|\psi\rangle = 1$

Thus, without loss of generality

$$|\psi\rangle = \cos\left(\frac{\theta}{2}\right)|0\rangle + e^{i\varphi}\sin\left(\frac{\theta}{2}\right)|1\rangle$$

where $0 \leq \theta \leq \pi$ and, $0 \leq \varphi \leq 2\pi$.

This connection between S^1 complex order parameter and the Bloch sphere S^2 is not unrelated to the __Complex Hopf Fibration__.

n-vector		BEC	MAG
C: 2-vector	$S^3 \rightarrow$	$S^1 \rightarrow$	S^2

BULK dimensions

RESTRICTED dimensions

How does one operate on a qubit ?

Changing the state of a qubit amounts to performing Rotations on the Bloch sphere.

We consider only rotations of the qubit state that keep its length invariant.

Such a rotation of a qubit can be easily accomplished by employing its representation as an SU(2) 2x2 unitary matrix.

Rotation on the Bloch sphere, about an axis $\hat{n} = (n_x, n_y, n_z)$ has the form:

$$U_{\hat{n}}(\theta) \equiv e^{-\hat{\imath}\frac{\theta}{2}\hat{n}\cdot\vec{\sigma}} = \sigma_0 \cos\left(\frac{\theta}{2}\right) - \hat{\imath}(\hat{n}\cdot\vec{\sigma}\,)\sin\left(\frac{\theta}{2}\right)$$

$\sigma_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ is the identity matrix

A qubit rotation by angle θ about an axis $\hat{n}$: $\mathrm{M}_{\mathrm{q}'} = U_{\hat{n}}(\theta)\cdot \mathrm{M}_{\mathrm{q}} \cdot U_{\hat{n}}^{\dagger}(\theta)$

[1] Yepez, Jeffrey. "Lecture notes: Qubit representations and rotations." 2013.

What are the limits on Quantum Computation ?

It is most important KEEP QUANTUM COHERENCE for very long times.
This is why superconducting qubits (*macroscopic quantum phenomena*) are so attractive.

DECOHERENCE of a TLS, exponential decay of coherence in time, is caused by [2]: (1) coupling between the qubits (in memory) and the environment, (2) coupling between qubits (in register) and the control-system gate operations.

These factors are important to consider in actual qubit implementations which are based on Josephson junction chains [3] that exist in (1D+1t).

[2] Karasawa, Tokishiro, and Masanao Ozawa. "Conservation-law-induced quantum limits for physical realizations of the quantum NOT gate." *Physical Review A* 75.3 (2007): 032324.
[3] Gladchenko, Sergey, et al. "Superconducting nanocircuits for topologically protected qubits." *Nature Physics* 5.1 (2009): 48.

What is a Universal Quantum Computer ?

A universal quantum computer, or quantum Turing machine (QTM), provides a simple model to capture all of the power of quantum computation.

Quantum computers are constructed from quantum building blocks, namely qubits and quantum gates.

Just as qubits are the generalization of classical bits, quantum gates are the generalization of classical logic gates.

A whole `universal' set of gates is needed for `universality' of quantum computation – that is, networks with any possible quantum computation property][4,5]!

[4] D. Aharonov, A simple Proof that Toffoli and Hadamard are Quantum Universal, (quant-ph/0301040)
[5] Kitaev, A. Yu. "Quantum computations: algorithms and error correction." *Russian Mathematical Surveys* 52.6 (1997).

It is also possible to get universality of gates from a single two-qubit gate.

Two-qubit gates, that are allowed to act on qubits within arbitrary range, serve as universal quantum gates for quantum computation [6].

in the computational basis $|\psi\rangle = a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle$

An example of a two-qubit gate:

$$G = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 0 & 0 & 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$$

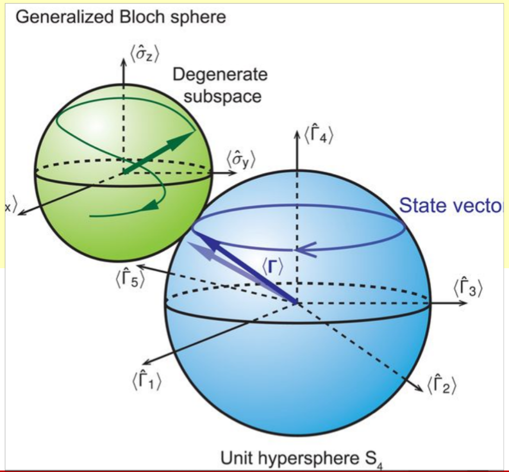
JJ of Quaternion (S^3) ordered system would be best!

Q: 4-vector $S^7 \to S^3 \to S^4$

$$\psi = |\psi| e^{\tau\theta}$$

Quaternion JJs form naturally in 4D/(3D+1t)

Representation on generalized Bloch sphere (S^4) [7].



[6] Shepherd, Daniel J., Torsten Franz, and Reinhard F. Werner. "Universally programmable quantum cellular automaton." *Physical review letters* 97.2 (2006): 020502.
[7] Sugawa, Seiji, et al. "Second chern number of a quantum-simulated non-abelian yang monopole." *Science* 360.6396 (2018): 1429-1434.

How can we push qubit technology even further ?

Three-qubit gates also allow for universality [8]

O: 8-vector $S^{15} \rightarrow S^{7} \rightarrow S^{8}$

$\{ |000\rangle, |001\rangle, |010\rangle, |011\rangle, |100\rangle, |101\rangle, |110\rangle, |111\rangle \}$

These are the only three meaningful Hopf fibrations and Lie algebra domains -- which suggests that one- two- and three-qubits are quite special [9].

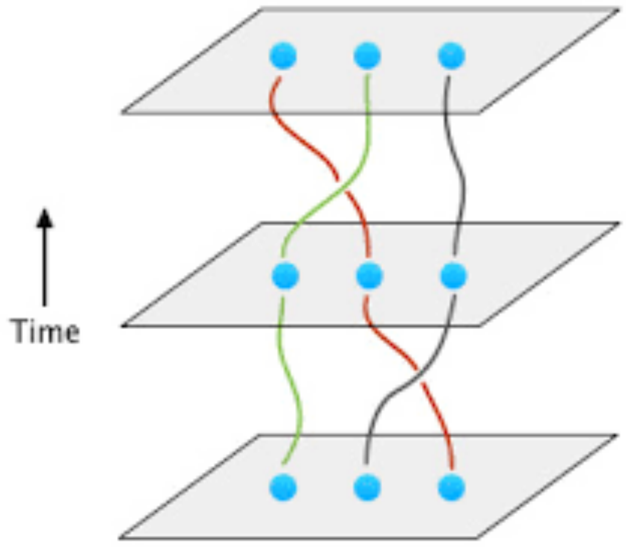
[8] Mosseri, “Two and three qubits geometry and Hopf Fibrations,” arxiv quant-ph/0310053.
[9] Nieto, Juan Antonio. "Duality, matroids, qubits, twistors and surreal numbers." *arXiv preprint arXiv:1810.04521* (2018).

What about a *Topological Quantum Computer* ?

A topological quantum computer employs qubits and quantum gates based on *topological structures* that are fault-tolerant by nature [10]!

In the simplest case, logic gates in the topological quantum computer are based on 2D ANYON quasiparticles, that form braids in 3D=(2D+1t) spacetime!

Topological quantum computers, based on quantum braids, are more stable than traditional quantum computers that use trapped quantum particles.



[10] Kitaev, A. Yu. "Fault-tolerant quantum computation by anyons." *Annals of Physics* 303.1 (2003): 2-30.

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